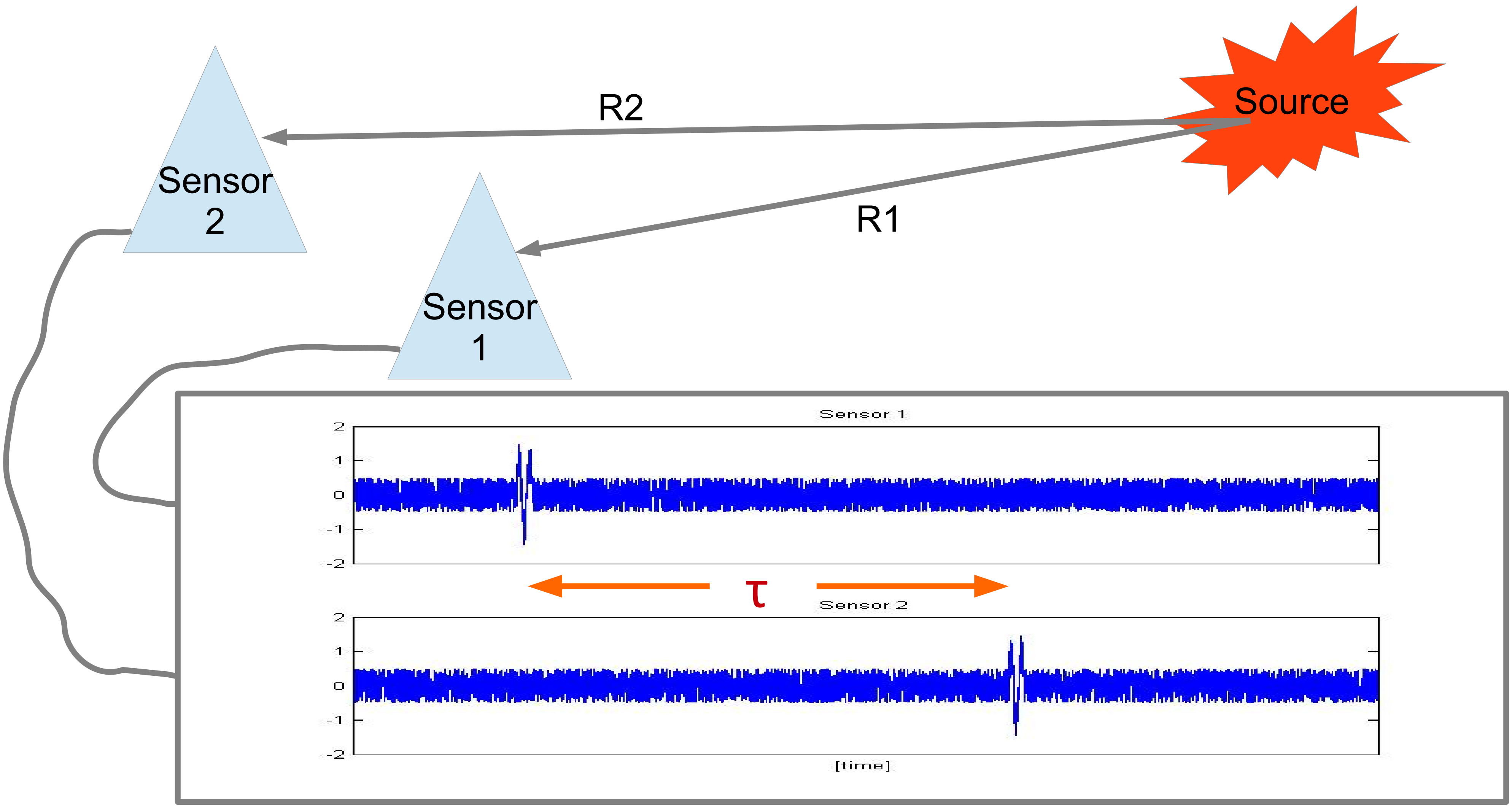
White paper:

Extracting the time-shift between 2 signals using the phase in frequenct domain



Introduction

- An 'event' signal from a source is recorded in 2 sensors.
- Location of the sensors is known.
- The event signal is identical in the 2 sensors, but shifted in time.
- We can find the time-shift by the correlation function:

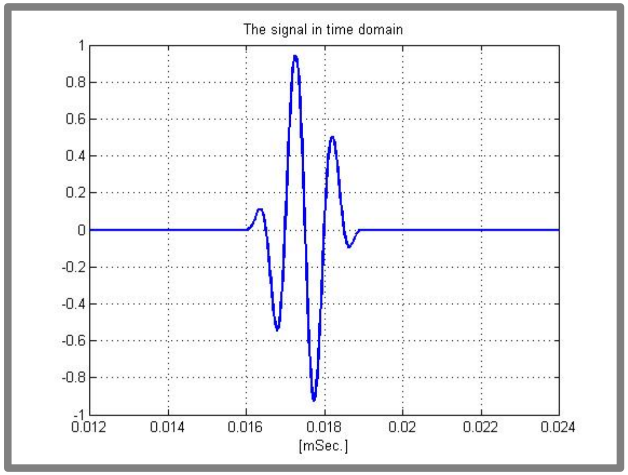
$$g(\tau)=\int S_1(t)\cdot S_2(t+\tau)\,dt$$

- But instead, we will use Fourier transform (DFT), where a time-shift transforms into a phase:

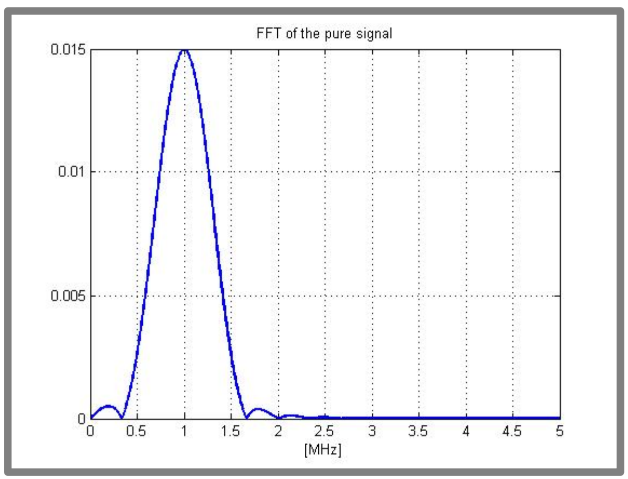
$$f(t-\tau)\Leftrightarrow e^{-i\omega\tau}\hat{f}(\omega)$$

- In this paper we are using synthesized signals for demonstration, as will be explained...
- In general, time-shift translates into a difference in the source-sensor distance by $\Delta x=\frac{\tau}{c}$ with c the propagation speed.

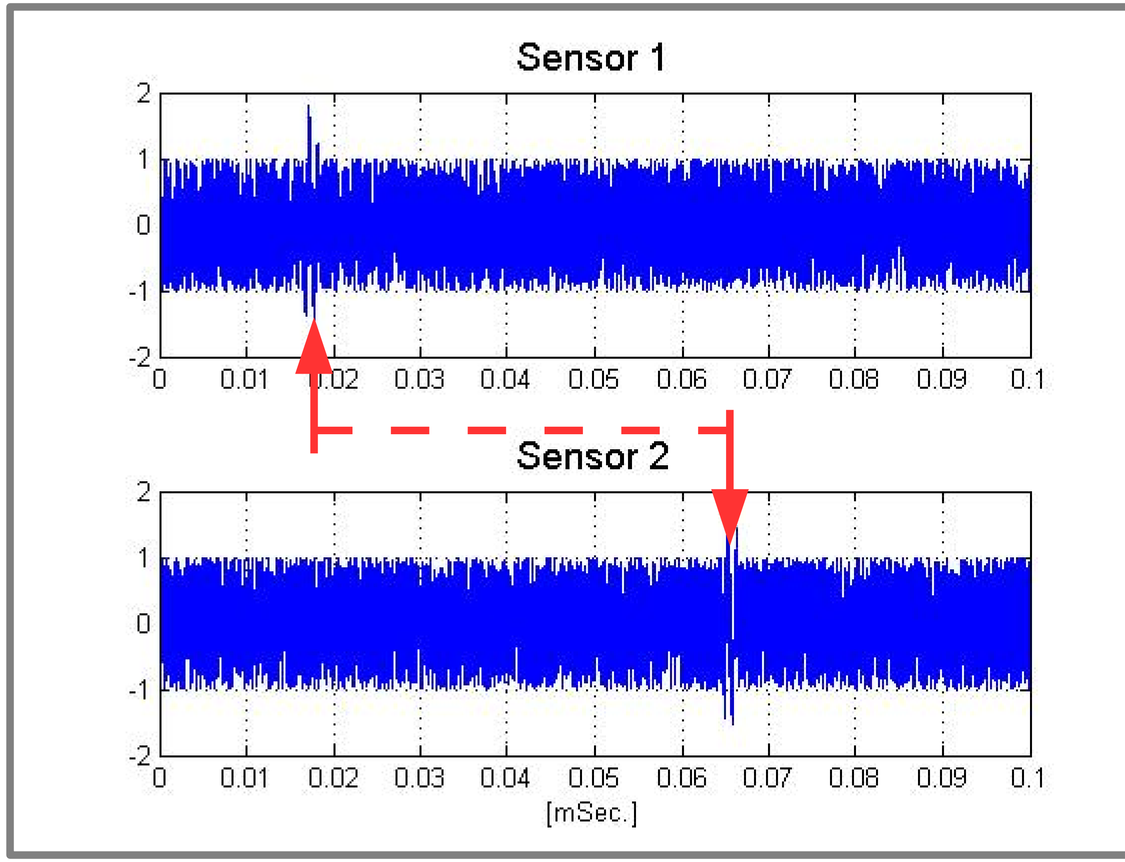
The Synthesized signals



This is the pure event signal



Spectrum of the pure event signal:
F (center) = 1.0 MHz,
Bandwidth ~ 0.7 MHz



Now we put the event signal in a random noise.
S1 and S2 represent the signals at sensor 1 and sensor 2.
There is a shift of 48 uSec. between the event signals.
The noise in the two sensors is uncorrelated.
SNR = 5 dB.

The problem:

- Assuming the same event is recorded at different times in 2 sensors: S1, S2
- The signal is covered by random uncorrelated noise
- We know something about the characteristics of the event signal: central frequency, bandwidth
- What is the time shift of the event signal in the 2 sensors?

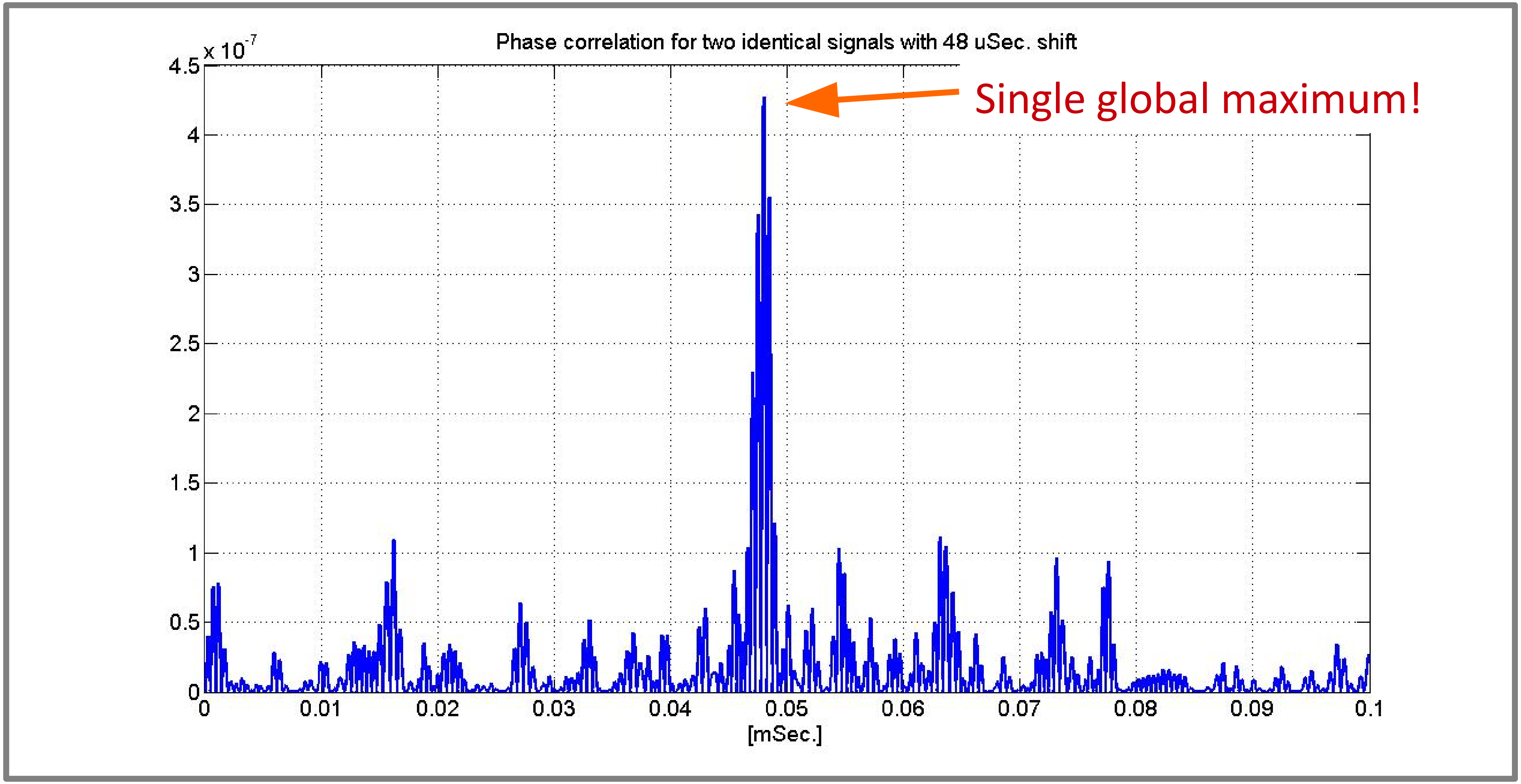
Algorithm:

- Get the DFT of each signal: $S_1(t) \Rightarrow \hat{S}_1(f),\, S_2(t) \Rightarrow \hat{S}_2(f)$
- Filter the signals in frequency domain, leaving only amplitudes within bandwidth around center frequency, all the rest amplitudes are set to zero. (*note: each amplitude appears twice, first at index k and second its complex conjugate at index N+1-k (be careful here!)*) $\hat{S}_i(f) \Rightarrow \hat{s}_i(f)$
- Multiply them term by term while the second is complex conjugated:

$$\hat{C}(f) = \hat{s}_2(f) \cdot conj\left(\hat{s}_1(f)\right)$$

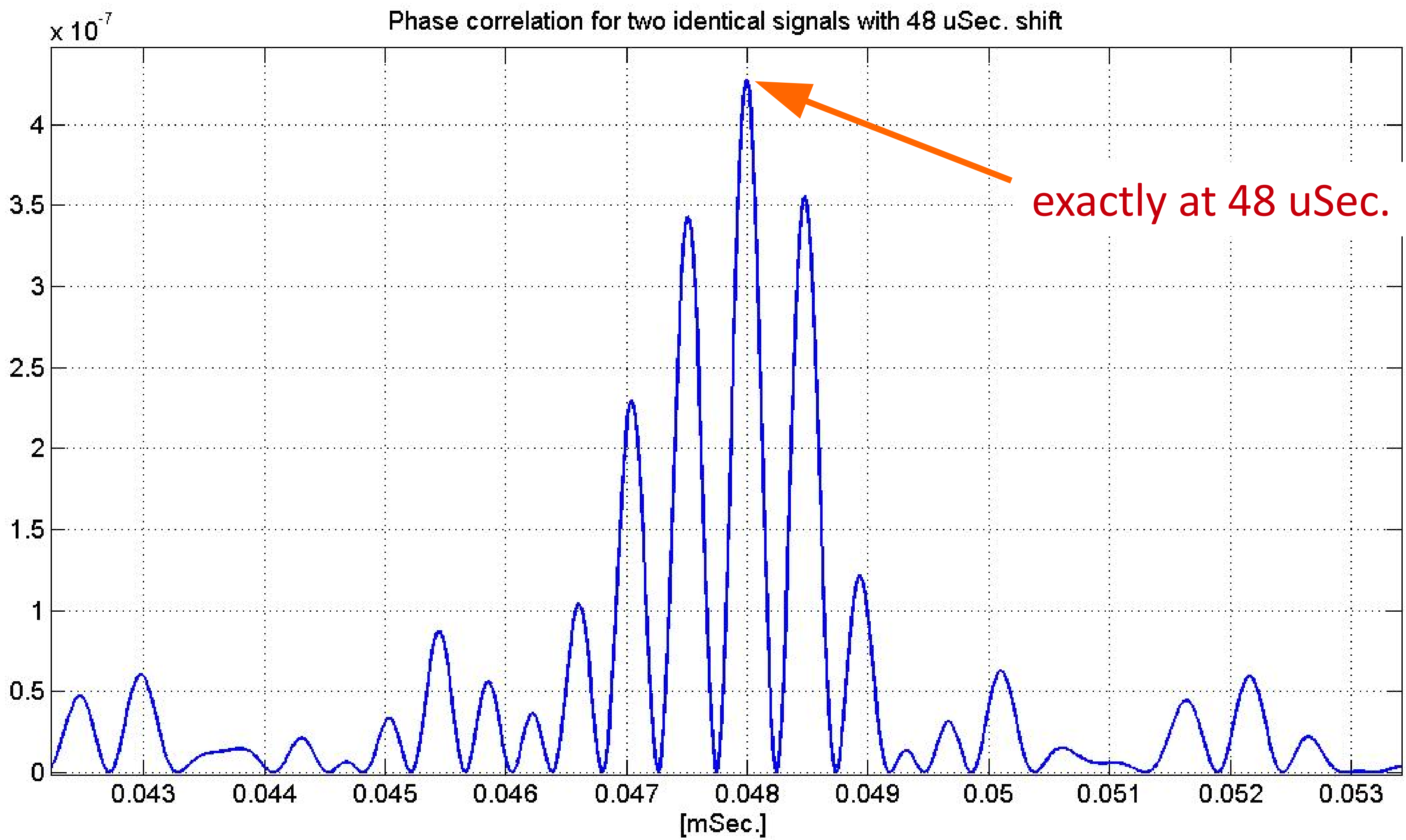
- Divide by the norm: $\hat{c}(f) = \dfrac{\hat{C}(f)}{|\hat{C}|} \quad , \quad where \quad |\hat{C}| = \sqrt{\sum_k \hat{C}_k \cdot conj\left(\hat{C}_k\right)}$
- Inverse transform to get the correlation function: $c(t) \Leftarrow \hat{c}(f)$
- Plot c(t) or better c^2(t). The time-shift is where c gets maximum.

Results



Phase correlation for two identical signals with 48 uSec. shift
x 10^-7
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
[mSec.]
Single global maximum!

Results



Phase correlation for two identical signals with 48 uSec. shift
x 10^-7
0
0.5
1
1.5
2
2.5
3
3.5
4
0.043
0.044
0.045
0.046
0.047
0.048
0.049
0.05
0.051
0.052
0.053
[mSec.]
exactly at 48 uSec.